$$\phi = -\frac{m}{r}\delta r = -\frac{m\delta s}{r}\cos\theta$$

$$\phi = \frac{\mu\cos\theta}{r} = \frac{\mu}{r}\cos\theta \qquad \mu = m\delta s$$

Let $\frac{\mu}{r}\cos\theta = c$ where c is cont.

$\Rightarrow \mu r\cos\theta = cr^2$

$\Rightarrow x^2+y^2 = \frac{\mu}{c}x$

$\Rightarrow \mu x = c(x^2+y^2)$

The curve ϕ = constant. this represent the circles touching the y-axis at the origin

We know that

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r}$$

$$\Rightarrow \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -\frac{\mu\cos\theta}{r^2}$$

$$\frac{\partial\psi}{\partial\theta} = -\frac{\mu\cos\theta}{r}$$

$$\Rightarrow \psi = -\frac{\mu}{r}\sin\theta$$

Constant of integration vanishes as θ and ψ both Vanish.

$$W = \phi + i\psi = \frac{\mu}{r}\cos\theta - i\frac{\mu}{r}\sin\theta$$

$$= \frac{\mu}{r}(\cos\theta - i\sin\theta)$$

$$= \frac{\mu}{r}e^{-i\theta}$$

therefore a source of strength place outside the circle $z=f$ will give Liesed and equal source $+m$ at an inverse Point and a sink $-m$ at an origin.

(iii) A sink of strength $-m$ at an origin $z=0$

$$w = m\log z - m\log(z-f) - m\log\left(z-\frac{a^2}{f}\right)$$

Put $z = ae^{i\theta}$

$$w = m\log(ae^{i\theta}) - m\log(ae^{i\theta}-f) - m\log\left(ae^{i\theta}-\frac{a^2}{f}\right)$$

$$= m\log(a+i\theta) - m\log[(a\cos\theta - f) + ia\sin\theta] - m\log\left(a\cos\theta - \frac{a^2}{f}\right) + ia\sin\theta)$$

$$w = \phi + i\psi$$

$$\psi = m\theta - m\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta - f}\right) - m\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta - \frac{a^2}{f}}\right)$$

Note → $\boxed{\log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{y}{x}}$

$$m\theta - m\tan^{-1}\left[\frac{\frac{a\sin\theta}{a\cos\theta - f} - \frac{a\sin\theta}{a\cos\theta - a^2/f}}{1 + \frac{a\sin\theta}{(a\cos\theta - f)}\cdot\frac{a\sin\theta}{(a\cos\theta - a^2/f)}}\right]$$

$$= m\theta - m\tan^{-1}(\tan\theta)$$

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$$w = \log\left(z - \frac{a^2}{z}\right)$$

$$= \log\left(\frac{z^2 - a^2}{z}\right)$$

$$= \log (z-a)(z+a) - \log z$$

$$w = \log (z-a) + \log (z+a) - \log (z-0)$$

Complex Potential shows that there are

(i) Two sinks of unit strength at the Point $(a,0)$ and $(-a,0)$

(ii) one source of unit strength at an origin

$$w = \phi + i\psi = \log[(x-a)+iy] + \log[(x+a)+iy] - \log(x+iy)$$

Equating real and Imaginary Part we have

$$\psi = \tan^{-1}\frac{y}{x-a} + \tan^{-1}\frac{y}{x+a} - \tan^{-1}\frac{y}{x}$$

or $$\psi = \tan^{-1}\frac{\frac{y}{x-a} + \frac{y}{x+a}}{1 - \frac{y}{x-a}\cdot\frac{y}{x+a}} - \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{y(x+a) + y(x-a)}{x^2 - y^2 - a^2} - \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{2xy}{x^2 - y^2 - a^2} - \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{\frac{2xy}{x^2-y^2-a^2} - \frac{y}{x}}{1 + \frac{2xy}{x^2-y^2-a^2}\cdot\frac{y}{x}}$$

$$= \tan^{-1}\frac{2x^2y - y(x^2-y^2-a^2)}{x(x^2-y^2-a^2) + 2xy}$$

$$= \tan^{-1}\frac{x^2y + y^3 + a^2y}{x^3 + xy^2 - a^2x}$$

$$= \tan^{-1}\frac{y(x^2+y^2+a^2)}{x(x^2+y^2-a^2)} = k \text{ (constant)}$$

$$\Rightarrow \frac{y(x^2+y^2+a^2)}{x(x^2+y^2-a^2)} = \tan k = c = \text{constant}$$

The Stream lines are given

$\psi = \text{constant}$

$\Rightarrow \theta_1 + \theta_2 = \text{constant}$, $m \neq 0$

If the Point P is Y-axis then

$\theta_1 + \theta_2 = \pi$

Thus there is no fluid flow across the line YOY'

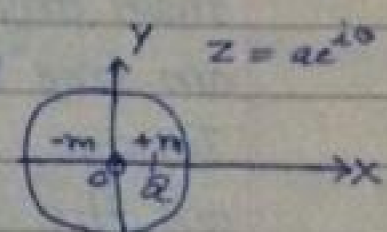
Hence it is stream line.

—x—

Image of a source with regard to a circle:-

Consider a source of $+m$

Place outside the circle of Radial a at a distance known as f



By O the. the complex Potential is given by

$OP \cdot OQ = a^2$ $\quad W = -m\log(z-f) - m\log\left(\frac{a^2}{z} - f\right)$

$\Rightarrow OQ = \frac{a^2}{f}$ $\quad$ By circle theorem

$$W = -m\log(z-f) - m\log\left(\frac{a^2}{z} - f\right) + m\log(-f)$$

or $W = -m\log(z-f) - m\log(a^2 - zf) + m\log z$

$= \cancel{m\log z - m\log(z-f) - m\log\{z(-f)}$

$$W = -m\log(z-f) - m\log\left[(-f)\left(z - \frac{a^2}{f}\right)\right] + m\log(-f) + m\log z$$

$$= m\log z - m\log(z-f) - m\log\left(z - \frac{a^2}{f}\right) - m\log(-f)$$

↳ cont.

$$W = m\log z - m\log(z-f) - m\log\left(z - \frac{a^2}{f}\right)$$

$$\boxed{W = m\log z - m\log(z-f) - m\log\left(z - \frac{a^2}{f}\right)}$$

Here const - term $-m\log(-f)$ is Adjused in W.

the perfect differential

$$M = -\frac{1}{2}\rho \int_c q^2 \left(x \frac{dx}{ds} + y \frac{dy}{ds} \right) ds$$

$$= -\frac{1}{2}\rho \int_c q^2 (x\cos\theta + y\sin\theta)\, ds$$

$$= -\frac{1}{2}\rho\, (\text{R.P. of}) - \int_c \frac{1}{2}\rho q^2 (x+iy)(\cos\theta - i\sin\theta) ds$$

$$= \text{R.P. of } -\frac{1}{2}\rho \int_c q^2 (x+iy)(\cos\theta - i\sin\theta) ds$$

$$= -\frac{1}{2}\rho \int_c q^2 z\, e^{-\theta i} ds$$

$$= -\frac{1}{2}\rho \int_c z\, (q^2 e^{-2\theta i})(e^{\theta i})\, ds$$

$$\Rightarrow M = \text{R.P. of } -\frac{1}{2}\rho \int_c z \left(\frac{dw}{dz}\right)^2 dz$$

Where M is the hydrodynamic moment action on the body. it is taken in the clock wise direction.

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